WELCOME

The Rise of Abstract Algebra

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Outline

Abstract Algebra

Origin of Set Theory

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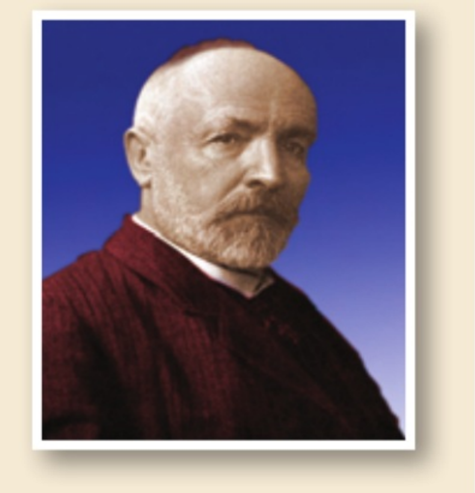

Origin of Set Theory

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- He worked on certain kinds of infinite series particularly on Fourier series
- Most mathematicians accept set theory as a basis of modern mathematical analysis
- Cantor's work was fundamental to the later investigation of Mathematical logic.

Set Theory

Introduction to Sets

Definition

A set is a collection of well defined objects or things.

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on.

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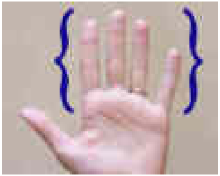

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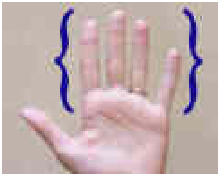

So it is just things grouped together with a certain property in common.

Introduction to Sets

Notation for Examples

$\{$ socks, shoes, watches, shirts, $\ldots\}$ - For Example 1

Introduction to Sets

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{ socks, shoes, watches, shirts, ... } - For Example 1

{ index, middle, ring, pinky } - For Example 2

Introduction to Sets

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Introduction to Sets

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The first set { socks, shoes, watches, shirts, . . .} we call an infinite set,
the second set { index, middle, ring, pinky } we call a finite set.

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For Example

$$A = \{a, e, i, o, u\}$$

Here A denotes the set of vowels, and a, e, i, o, u is an element of the set A.

Formal Definitions

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The main property of a set in mathematics is that it is well-defined. This means that given any object, it must be clear whether that object is a member (element) of the set or not. The objects of a set are all distinct, i.e., no two objects are the same.

Algebra

The term algebra usually denotes various kinds of mathematical ideas and techniques, more or less directly associated with formal manipulation of abstract symbols *and/or* with finding the solutions of an equation.

Group Theory

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(3) The collection of districts in Tamil Nadu.

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An operation combines members of a set.

I can use painting as an Example

Let's imagine we have the set of colors { red, green, blue }.

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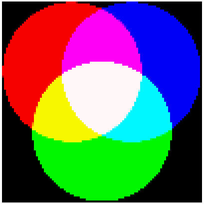

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- $4 - 4 = 0$

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$$* : S \times S \to S \text{ by } (a, b) \to a * b$$

That is

$$a, b \in S \Longrightarrow a * b \in S$$

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- ❋ The group contains inverse

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If we have two elements in the group, a and b, it must be the case that $a * b$ is also in the group. This is what we mean by closed.

Formal Statement

For all elements a, b in G, $a * b$ is in G

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For all a, b and c in G, $a * (b * c) = (a * b) * c$

The group contains inverses

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For all a in G, there exists b in G, such that $a * b = e$ and $b * a = e$.

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Formal Statement

There exists an e in the set G, such that $a * e = a$ and $e * a = a$, for all elements a in G

Formal Definition

A non-empty set G,

Formal Definition

A non-empty set G, together with an operation $*$

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- **Identity axiom** : There exists an element $e \in G$ such that $a * e = e * a = a, \forall\, a \in G$.
- **Inverse axiom** : $\forall\, a \in G$ there exists an element $a^{-1} \in G$ such that $a^{-1} * a = a * a^{-1} = e$.

Commutative property

A binary operation $*$ on a set G is said to be commutative, if

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If a group satisfies the commutative property then it is called an abelian group or a commutative group, otherwise it is called a non - abelian group.

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(2) The Greek mathematician Diophantus has traditionally been known as the "*fatherofalgebra*" but in more recent times there is much debate over whether $al - Khwarizmi$, who founded the discipline of $al - jabr$, deserves that title instead.

Abstract Algebra means

Algebra in Research

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The Rise of abstract Algebra

The international character of the subject is seen in the fact that algebra the two most revolutionary contributions, in 1843 and 1847,were made by Mathematician who taught in Ireland.

❁❁❁ Time to Interact ❁❁❁


Thank You